SUMMARY A computer model was developed to describe regional cerebral blood flow and tissue oxygenation with autoregulation during focal ischemia produced by occlusion of the middle cerebral artery (MCA). This steady state model described the distribution of blood flow in the cerebral arterial system including the circle of Willis as well as the pial arterial anastomoses, and included a simplified form of autoregulation based on the local control of pressure and flow in the pial and intracerebral arteries, respectively. Preliminary simulation studies with the model yielded the following results. Less effective autoregulation was predicted by the model at low blood pressure in focal ischemia. Passive dilatation of the pial vasculature produced a leftward shift in the autoregulatory curve. Simulations with occlusion of the MCA revealed the ultimate importance of the pial anastomoses in providing adequate blood and oxygen supply in the ischemic territories including the specially vulnerable lenticulostriate area. The volume of the ischemic (pO_2 < 1 mmHg) brain tissue in the MCA-cortex estimated by using a concurrent Krogh cylinder model was 50% when the pial anastomoses were 80 μm in diameter and the ischemic area disappeared at 170 μm diameter. With relatively small anastomoses (< 200 μm) the model demonstrated intracerebral steal during intracerebral vasodilation. Passive dilatation of the pial arteries including the pial anastomoses caused the steal to disappear and to reverse. These results suggest that both autoregulatory shift and steal reversal can be explained by passive dilatation of the pial vasculature.

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cylinder model. Thus, dependence of regional cerebral blood flow and oxygen supply on the pial collateral circulation during focal ischemia produced by long-term MCA occlusion could be studied.

**Description of the Model**

The hemodynamic model consisted of the two internal carotid arteries (ICA), the basilar artery (BAA), the circle of Willis, the anterior (ACA), middle (MCA) and posterior (PCA) cerebral arteries, and the lenticulostriate (LSA) arteries. A schematic diagram of the model is displayed in figure 1. Pial anastomotic connections between the main cortical arteries were represented by lumped segments (AM & PM) connecting the anterior and posterior cerebral arteries to the middle cerebral artery respectively. Branches of the ACA, MCA, and PCA were lumped into preanastomotic (pial) and postanastomotic (intracerebral) segments. The postanastomotic segments also included the capillaries and veins. The lenticulostriate vessels were lumped into a single vessel segment.

The model thus comprised 32 vessels and lumped segments connected to each other at 17 branch points. In order to determine hydrodynamic resistance of the vessels, diameter and length measurements of the major cerebral arteries were performed on several monkey brains perfused with formalin at normal arterial pressure. Cerebral arterial casts were also prepared by infusion of plastic* with subsequent corrosion of the brain in concentrated HCl. Although the fixation techniques might have altered somewhat the luminal diameters, the more important ratios of diameters were probably not influenced significantly. Diameters and lengths of the vessels used for computer simulation are shown in table 1. For vessels having definite length, hydrodynamic resistance was calculated by $R = \frac{8 \eta L}{\pi r^4}$ ($\eta = 3.0$ cp). Hydrodynamic resistance of the lumped pial arteries was set equal to 26% of the total cerebrovascular resistance (CVR), while that of the postanastomotic vessels was set equal to 57% of CVR.\(^4\) The CVR of the monkey was calculated from an arterial pressure of 120 mmHg and a total cerebral blood flow of 25 ml/min.

Lengths of the basilar and internal carotid arteries listed in Table 1 are not physiological but effective lengths, which were adjusted to achieve 17% of the total CVR for the large arteries.\(^5\) In steady flow models it is not the individual values of length and diameter but their ratio $L/r^4$ (i.e. the resistance) that is impor-

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*Batson's \#17 Anatomical corrosion compound diluted 2:1 with J.B. 4 solution a. (Polyscience, Inc.)

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**Table 1** Geometrical Data of the Cerebral Arterial Segments of the Monkey

<table>
<thead>
<tr>
<th>Arterial segment</th>
<th>Diameter/mm</th>
<th>Length/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal Carotid</td>
<td>1.1</td>
<td>170.0*</td>
</tr>
<tr>
<td>Basilar</td>
<td>0.8</td>
<td>170.0*</td>
</tr>
<tr>
<td>Anterior Cerebral</td>
<td>0.6</td>
<td>8.0†</td>
</tr>
<tr>
<td>Middle Cerebral</td>
<td>0.8</td>
<td>1.0‡</td>
</tr>
<tr>
<td>Posterior Cerebral</td>
<td>0.7</td>
<td>4.0‡</td>
</tr>
<tr>
<td>Lenticulostriate</td>
<td>0.2</td>
<td>—</td>
</tr>
<tr>
<td>Anterior Communicating</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td>Posterior Communicating</td>
<td>0.2</td>
<td>5.0</td>
</tr>
</tbody>
</table>

*Effective lengths.
†Length of the proximal segment.
tant. The resistance value assigned to the large arteries was obtained in cats, and is likely greater than that of the monkey due to the carotid rete. For monkeys, however, we found no adequate data in the literature.

Since pial anastomotic vessels form a complex network on the cortical surface, an effective length was assigned to these segments as well. The average number of the anastomotic vessels (7 between ACA and MCA), and 4 between PCA and MCA) was included in the calculation of the resistance.

Autoregulation of flow in the cerebrovascular model was based on the integral control of local perfusion pressure and flow in the pial and intracerebral segments. Pial arteries play an important role in autoregulation during alterations in arterial blood pressure. Therefore, resistance of the preanastomotic (pial) segments was controlled by the pressure at the outlet of these segments. This idea was suggested by the theory of sequential myogenic control, however, the specific mechanism of the control is not defined explicitly in the present work. Postanastomotic (intracerebral) resistances were controlled in the model by local blood flow rates. Since approximately 44% of the intracerebral resistance (25% of CVR) belongs to the capillaries and veins, which is approximately constant, only the remaining 56% (32% of CVR) that belongs to the intracerebral arteries was allowed to change. In complete vasodilatation resistance of pial and intracerebral segments was allowed to decrease to 1/16 of the initial value which means a twofold increase in internal diameter. The resistance of large arteries was kept constant.

Assuming steady, laminar flow and writing linear pressure-flow relations for the 32 vessel segments and the appropriate flow balances for the 17 branch points, the system of 49 linear equations with 49 unknowns (32 flows, 17 pressures) was solved on digital computer. For the simulation of autoregulation an iterative procedure was used. Using the pressure and the flow values obtained from the previous solution of the equations, the appropriate segmental resistances were modified and used for the next approximation. The iterative solutions were always convergent within 10–15 steps.

In order to characterize the severity of cerebral ischemia in the areas having reduced blood flow, oxygen partial pressure in the tissue was calculated using a steady Krogh cylinder model with concurrent flow. Neglecting axial diffusion, radial distribution of oxygen partial pressure in the tissue at each axial position along the capillary was calculated from the equations:

\[ S = S_a - \frac{A(R_c^2 - R_i^2)}{K_r R_i^2} z, \quad (z = O, L) \]  
\[ P_b = \left( \frac{K_r S}{1 - S} \right) K_r, \]  
\[ P(r) = P_b - 2K_r A R_i^3 \left( \ln R_i / R_v \right) + K_r A (r^2 - R_i^2), \quad r = (R_p, R_c) \]

where the following nomenclature has been used:
- \( S \): oxygen saturation of blood in the capillary
- \( S_a \): arterial oxygen saturation
- \( A \): oxygen consumption in the gray matter
- \( R_c \): radius of the oxygen consuming tissue cylinder
- \( R_i \): capillary radius
- \( v \): average blood velocity
- \( z \): axial coordinate
- \( L \): capillary length
- \( P_b \): average \( pO_2 \) in the blood
- \( P(r) \): \( pO_2 \) in the tissue at the radial coordinate \( r \)
- \( M_i = K_r \): constants

In the present calculations, resolution of 50 steps in the radial and 10 steps in the axial directions was used. In order to determine certain parameters of the model frequency distribution of \( pO_2 \) was generated using the calculated \( pO_2 \) values for normally perfused tissue and compared to a histogram of experimental data measured in the rabbit cortex. Normal oxygen consumption and capillary length were obtained from the best fit of \( pO_2 \) histograms keeping the arterial and venous \( pO_2 \) at 95 and 35 mmHg, respectively (fig. 2). Other parameters were the same as used by Reneau et al., Parameter values used for the present simulation are listed in table 2.

For normal flow the \( pO_2 \) was about 1 mmHg in the so-called lethal corner of the Krogh cylinder (fig. 3). Under ischemic conditions the oxygen consumption was assumed to follow zero order kinetics. That is, \( A \) was homogeneous and constant in the tissue cylinder except in the points where \( pO_2 \) was less than 1 mmHg, where \( A \) was set to zero. Accordingly, at each axial position \( R_i \) was reduced to exclude the non-consuming tissue areas. The tissue \( pO_2 \) values were then recalculated and a new radial \( R_i \) established. Final \( pO_2 \) distribution was obtained by repeating this procedure iteratively until the results became stable.

Figure 3 shows the shape and location of anoxic (\( pO_2 < 1 \) mmHg) areas in the Krogh cylinder at different reduced flow rates. The ITV (ischemic tissue volume) index was defined as the relative volume of the anoxic tissue. Although ITV was not related directly to the gross volume of ischemic brain tissue it was accepted as a local measure of the severity of ischemia.

**Results**

Figure 4 shows cerebral autoregulation curves obtained by computer simulation. Since we were interested in hypotensive cases which accompany MCA occlusion, only the low pressure part of the autoregulation has been studied. The model demonstrates that the total cerebral blood flow (CBF) decreases when the arterial pressure is reduced below 60–70 mmHg. During unilateral occlusion of the MCA the autoregulation is partially damaged. Total CBF drops initially to 80–90% of the preocclusion value, which agrees well with the experimental findings. With autoregulation, CBF reaches a slightly higher level. A leftward shift in the autoregulatory curve is also demonstrated, which occurs when maximum diameter of the pial arteries is increased twofold.
Regional autoregulation in the MCA territory is demonstrated in figure 5. Occlusion of the MCA leads to total disappearance of autoregulation in the ischemia area. Regional blood flow (rCBF) decreases to 42% of the preocclusion value at 120 mmHg mean arterial pressure. The post occlusion rCBF, however, depends largely on the actual diameter of the pial anastomoses. For example, if the anastomotic diameter varies over the range of 80-140 μm, blood flow in the MCA territory may decrease initially to 10-67% of the preocclusion value. Simultaneously, the ITV index varies in the interval of 2-40%, depending on the anastomotic diameter.

Influence of the actual anastomotic diameter on regional blood flow and oxygen supply in the specially important lenticulostriate area is demonstrated in figure 6. Oxygen supply is characterized by the average tissue pO₂ at the venous end of the Krogh cylinder — as a representative value of tissue oxygenation in the weakly supplied microregions, and by the relative ischemic tissue volume (ITV). rCBF and this average pO₂ increases and ITV decreases with increasing anastomotic diameters during MCA occlusion. In addition, average pO₂ is a linear function of the anastomotic diameter with a correlation coefficient of 0.98. An anastomotic diameter of at least 170 μm is necessary to remove ischemia completely from the tissue.

Based on the hypothesis of long-term pial vascular adaptation in focal ischemia, further computer simulations were run to investigate the dependence of rCBF on the anastomotic diameter as a function of time elapsed from the beginning of the MCA occlusion. The mechanism of the adaptational process was presumed

**TABLE 2 Parameters of the Krogh Cylinder Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₂</td>
<td>0.9573</td>
<td>K₁</td>
<td>0.204</td>
</tr>
<tr>
<td>A</td>
<td>19 × 10⁻⁴ ml·cm⁻³·s⁻¹</td>
<td>K₂</td>
<td>10³</td>
</tr>
<tr>
<td>R₁</td>
<td>30 μm</td>
<td>K₃</td>
<td>0.4545</td>
</tr>
<tr>
<td>R₂</td>
<td>2.5 μm</td>
<td>K₄</td>
<td>5.07 × 10⁸</td>
</tr>
<tr>
<td>ν</td>
<td>0.004 cm·s⁻¹</td>
<td>L</td>
<td>72 μm</td>
</tr>
</tbody>
</table>

Figure 3. Distribution of anoxic tissue (pO₂ < 1 mmHg) regions in the Krogh cylinder model at different reduced blood flow rates (10-80% of normal). ApO₂ = arterial oxygen partial pressure.

Figure 4. Dependence of cerebral blood flow (CBF) on the arterial pressure as simulated by the computer model. The lower limit of normal autoregulation (solid line) is shifted to the left in case of chronic pial vasodilation. Autoregulation is weakened during occlusion of the middle cerebral artery. D = average anastomotic diameter.
**Mathematical Simulation of Cerebral Blood Flow**

**Hudetz et al.**

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**Equation (4):**

\[
R(t) = R_n + (R_0 - R_n) \exp \left( -\frac{t}{T} \right),
\]

where:

- \( R(t) \) = pial anastomotic resistance at time \( t \)
- \( R_n \) = minimal anastomotic resistance
- \( R_0 \) = \( R(0) \) at the time of occlusion
- \( T \) = time constant of the adaptation.

---

**Figure 5.** Dependence of regional blood flow (rCBF) in the territory of the middle cerebral artery (MCA). Autoregulation is diminished in MCA occlusion in case of 120 µm average pial anastomotic diameter.

---

**Figure 6.** Influence of the diameter of pial anastomoses on regional blood flow (rCBF), ischemic tissue volume (ITV) and average tissue oxygen partial pressure around the venous end of capillary (pO₂) in the lenticulostriate area (LSA) during occlusion of the middle cerebral artery.

---

**Figure 7.** Time course of equilibration of regional blood flow (rCBF) in the MCA territory in case of different anastomotic diameters from 80 to 140 µm. \( T \) = time constant of pial vascular adaptation.

---

**Discussion**

Since the first attempt to model the hemodynamics of the circle of Willis by rigid plastic tubes, several physical, electrical, and more advanced computer models have been developed to simulate steady and pulsatile flow in the major cerebral arteries. In case of the occlusion of the MCA, however, the circle of Willis is of less significance because collateral circulation in the ischemic territory can be established through the pial anastomoses only. Since we have been...
interested primarily in the computer simulation of focal cerebral ischemia in response to MCA occlusion, the pial anastomoses connecting the branches of the anterior and the posterior cerebral artery with those of the middle cerebral artery were included as an essential component of the present cerebral circulation model.

Likewise, none of the above mentioned models have attempted to simulate cerebral autoregulation. Our preliminary studies revealed, however, that active alterations in the preanastomotic and postanastomotic arterial diameters may affect the pial collateral flow critically. Therefore, regional autoregulation was included in the present model. Autoregulation was represented as local pressure and flow control in the pial and intracerebral arteries, respectively. Although there is still controversy in the literature about the role of different local neural and humoral factors in cerebral autoregulation, the above scheme of segmental control seems to be justified.5

The integral control of pressure and flow used in the present model was an obvious oversimplification of the real cerebral autoregulation. Pressure at the pial anastomoses was kept constant until the preanastomotic resistance was reduced to minimum in contrast to the observation that pial artery pressure changes with the systemic pressure.5 Also, cerebrovascular resistance was constant below 50 mmHg in Figure 4 due to the maximal dilatation of both pial and intracerebral arteries, although cerebral arterioles were known to continue to dilate to pressures as low as 35 mmHg.20 However, in order to simulate the plateau and lower limit of the cerebral autoregulation the simplified scheme used for the control was satisfactory in the present model. The actual mechanism of pressure and flow regulation were not defined explicitly in the model. Overall cerebral autoregulation has been successfully simulated using \( pO_2 \), \( pCO_2 \), and \( pH \) as control signals,21 however, without knowing the specific segmental action sites of the controlling factors, their incorporation into a regional regulatory model was not reasonable. Nevertheless, the present mathematical model was able to simulate quasi-normal cerebral autoregulation (at least below 120 mmHg), autoregulatory shift in response to chronic dilation of the pial vasculature and disturbance of autoregulation during arterial occlusion.22 Since the gain of vascular control was not allowed to change in the model, this suggests that the breakdown of normal autoregulation is not necessarily related to reduced responsiveness of the cerebral arteries, but may be at least in part the consequence of the maximum dilatation of the arterial vasculature in the ischemic area.

In addition to the calculation of regional blood flow, the Krogh cylinder model made it possible to estimate the severity of ischemia by calculating the \( pO_2 \) distribution and the anoxic "microvolume" of a representative tissue cylinder. As a result of fitting the \( pO_2 \) frequency distribution to the measured data a very low \( pO_2 \) value, about 1 mmHg, was obtained in the lethal corner of the Krogh cylinder for normally perfused tissue. In accordance with the low \( pO_2 \) values the figure for normal oxygen consumption obtained from the best fit of the histograms was about two times higher than the figure (8.34 \( \times 10^{-3} \text{mlcm}^{-2}\text{s}^{-1} \)) used by Reneau et al.10 for cerebral gray matter. Also, the capillary length of 72 \( \mu \text{m} \) instead of 180 \( \mu \text{m} \) was obtained. This shorter average length of capillaries was confirmed by our recent morphologic studies on the cortex of monkey brains perfused with India ink after formalin fixation (unpublished). Regarding the figure obtained for the oxygen consumption, it cannot be excluded that a more sophisticated \( pO_2 \) model, such as the Krogh cylinder with countercurrent instead of concurrent flow,
might have predicted a lower consumption value with an equally good fit to the same pO₂ histogram.

Results of the present computer simulation have emphasized the importance of pial anastomoses in determining regional blood flow and oxygen supply of the ischemic cerebral tissue and thus the outcome of the ischemic cerebral injury in response to the occlusion of the middle cerebral artery. According to Blinkov and Glezer, large pial anastomoses of the monkey brain average 120 µm in diameter. This value gave 58% reduction in CBF in response to MCA occlusion. It is important to remember here, however, that only a virtual length was assigned to the anastomotic segments in the model. Pial anastomoses form a complex arterial network on the surface of the brain, a definite length thus could not be established. On the other hand, it was possible to estimate the overall hydrodynamic resistance of the anastomotic connections by comparing computer simulation results to experimentally measured data. Pial arterial pressure was found to decrease to 10–30 mmHg, or 20–30% of normal during MCA occlusion in the macaque. After adjusting the anastomotic resistance to the appropriate value, the model predicted a pial arterial pressure of 10–16 mmHg (18–32% of normal) in simulated MCA occlusion. Using the obtained anastomotic resistance and 120 µm for the anastomotic diameter, a virtual length of 3.5 mm was calculated and used for the computer simulation.

It is not only the anatomical variability of the diameter of anastomoses that we must consider but, also as we have mentioned earlier, a long-term adaptation of the pial vasculature. That is, if the pial arteries and anastomoses dilate to a certain extent during chronic MCA occlusion, regional and total CBF can be restored over a period of time. This does not occur necessarily in all experimental cases, but an approximate characteristic time of the flow restoration of 12 hours can be presumed. Supposedly, the vasodilatation in the ischemic region does not stop when the blood flow reaches the precocclusion value, but the process continues until the autoregulatory capacity is liberated and restored in that area. It is not possible to determine theoretically the limit of the adaptation, and the individual variation seems to be also rather large. Therefore, as a first approximation it was assumed that the arterial dilatation comes to a stop when the hemodynamics and autoregulation are completely restored in the previously ischemic area. In the present mathematical model a twofold increase in the pial arterial diameter with an initial anastomotic diameter of 120 µm was necessary to achieve complete restoration of autoregulation. This means that the final anastomotic diameter should be about 240 µm. Then, if the vascular adaptation is an exponentially decelerating process as a first approximation, the 12 h flow equilibration with the 240 µm final anastomotic diameter gives T = 18 h for the time constant. Using these figures, the computer simulation predicted an equilibration time of 6–16 hours if the anastomotic diameter varied in the interval of 80–140 µm. If not, only the initial anastomotic diameter but also the time constants of the adaptational process are different in each individual case, variation in the equilibration time can be even greater. Of course, the time course of pial vascular adaptation should be determined experimentally before conclusions can be drawn for the real cerebral circulation. Also, anastomoses of relatively small size may not always reach the maximal value of 240 µm in vivo. With the development of cerebral edema following MCA occlusion and consequent increase in intracranial pressure, the degree of cerebrovascular resistance at the capillary and venous levels will probably increase which may compromise the collateral circulation to the ischemic region and reduce the adaptational capacity of the pial vasculature. Consideration of such changes which are in competition with the adaptational process will be the subject of future improvement of the present hemodynamic model.

The present simulation predicts that pial arterial constriction in chronic hypertension would result in a larger volume of ischemia initially, as well as a longer time for adaptation of the collateral circulation, both of these resulting in a more severe clinical outcome. This is consistent with our recent experiments in the spontaneously hypertensive rat in which previously treated rats tolerated cerebral ischemia better than did untreated animals with the same arterial occlusion.

The model was also able to simulate intracerebral steal and inverse steal. These phenomena can be elicited experimentally by a strong vasodilator, like high arterial pCO₂. Since CO₂ acts primarily on the small cerebral vessels, and pial arterioles of similar size, the effect of CO₂ was simulated by maximal dilatation of the postanastomotic arterial segments. The present results suggest that the reversal of intracerebral steal can indeed be explained by the chronic dilation of the pial arteries including the pial anastomoses. With an initial anastomotic diameter of 120 µm a twofold increase in the pial diameters produced 20% increase in rCBF in response to acute intracerebral vasodilation during MCA occlusion. Recalling that the same extent of pial arterial dilation lowered the limit of autoregulation by 30–40 mmHg (fig. 3), this suggests that both the steal reversal and the autoregulatory shift can be the consequence of a chronic pial vascular adaptation in long-term cerebral ischemia.

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