Optimality Principles and Flow Orderliness at the Branching Points of Cerebral Arteries

Sandro Rossitti, MD, Jan Löfgren, MD, PhD

Background and Purpose: The cerebral arteries present an optimum blood flow/vessel radius relation. However, branch angles may vary widely in the cerebral arteries because the parametric optimization of branch angles is irrelevant in terms of energy cost. The position of the flow divider in extracranial arteries has been suggested to be optimum in flow orderliness. No data exist on the flow divider of cerebral arteries. Thus, we hypothesized that in the cerebral arteries the apex of the bifurcations, which is known to be the site of maximum hemodynamic stress in a vascular network, may normally lie in a nonoptimum position relative to the dividing flow streamline in the parent vessel, leading to disturbed laminar flow and increased vessel wall shear stress at the apical region despite the optimum blood flow/vessel radius relation. The objective of this study was to test our hypothesis.

Methods: We measured the branch angles and diameters of parent and branch segments of the anterior cerebral artery system from lateral projections to minimize the measurement error on angiographs chosen at random from normal sets. The position of the apex of the bifurcations in relation to the ostium of the parent artery (γ) and the ratio of the branch diameters (d2/d1) were compared. Optimum curves for these parameters were calculated by a mathematical model. In addition, the separation of flow streamlines according to γ was calculated for each bifurcation and related to the division of flow required by each branch according to the optimum blood flow/vessel radius relation.

Results: The data points on γ and d2/d1 and the separation of flow according to γ and the division of flow required by the branches were found to scatter around the optimum curves. However, a trend toward the theoretical optimum is discernible. The data points are suggested to be a random sample from a normal distribution around the optimum (.40 < P < .50).

Conclusions: The bifurcations of the cerebral arteries appear to be optimized to avoid increased hemodynamic stresses both globally and locally in the same manner as extracranial arteries. (Stroke 1993;24:1029-1032)

Key Words • arteriovenous malformations • cerebral aneurysms • cerebral arteries

The branching geometry of the internal carotid artery system was analyzed in a previous work to establish whether the cerebral arteries are optimized to avoid increased hemodynamic stresses.\(^1\) We concluded that the branching of the internal carotid artery system obeys the principle of minimum work, an optimization model for the growth and adaptation of arterial trees that establishes a balance between energy dissipation and energetic cost because of shear stress on the vessel wall and the volume of the blood and vasculature. The blood flow/vessel radius relation achieves an equilibrium when the radius (r) of the vessels is adjusted to the cube root of the volumetric flow (f), or f = kr\(^3\). Therefore, the diameter exponent defined by r\(_d\) = r\(_p\) + r\(_f\), where r\(_p\) is the radius of a parent artery and r\(_1\) and r\(_2\) are the radii of its branches, approximates n = 3. However, branch angles were found to vary widely, which is in conformity with other studies of the principle of minimum work showing that the parametric optimization of branch angles is irrelevant in terms of energy cost.\(^2\) Therefore, we hypothesized that the apex of the bifurcations, which is known to be the site of maximum hemodynamic stress in a vascular network, may normally lie in a nonoptimum position relative to the dividing flow streamline in the cerebral arteries, leading to disturbed laminar flow and increased shear stress on the vessel wall at the apical region despite the optimum blood flow/vessel radius relation. Earlier work on the flow divider of arterial bifurcations, conducted in diverse parts of the circulatory system of mammals, suggests that the position of the flow divider is optimum, but there are no data on the cerebral arteries.\(^3\) Our hypothesis has implications for understanding the formation of saccular aneurysms, which are rarely encountered in the extracranial arteries. Aneurysm formation might be related to branching characteristics of the cerebral arteries that locally increase the hemodynamic stress.\(^4,5\) Possibly, normal cerebrovascular geometry may be a risk factor in this context. The hypothesis demanded further analysis because the position of the apex of a bifurcation in relation to its branches may be the same, and perhaps the optimum one, in bifurcations with different branch angles. The objective of the present study was to compare the position of the flow divider of cerebral arteries with the division of flow given by the diameter exponent.

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From the Department of Neurosurgery, University of Göteborg, Sahlgrenska Hospital, Göteborg, Sweden.

Correspondence to Department of Neurosurgery, Sahlgrenska Hospital, S-413 45 Göteborg, Sweden (Dr Rossitti).
Materials and Methods

We used analog angiographs chosen at random from sets classified as normal from the archives of the Department of Neuroradiology, Sahlgrenska Hospital, Göteborg, Sweden. The methods of measurement have been detailed previously.\(^1\) In this study the measurements were done exclusively in the anterior cerebral artery system, which lies approximately in the sagittal plane, in this way minimizing the measurement error of the branch angles. Twelve vascular trees were analyzed up to the fourth branch generation. The branch angles of 70 bifurcations were measured from lateral projections. The branching points were traced on a transparent plastic sheet, and the angles were measured after photostatic magnification. Measurements of the diameters of parent and branch segments were done using a scale and hand lens. No correction was made for the magnification effect of the divergent x-ray beam. The unsharpness of vessel edge as the boundary is approached was minimized by choosing the picture with the best contrast resolution for each vessel from the angiographic series and by avoiding bifurcation measurements in which the parent vessel had an angiographic diameter of less than 1 mm. The measured parameters and the mathematical model\(^6\) are presented in Fig 1. The geometrical position of the bifurcation apex in relation to the ostium of the parent artery (\(\gamma\)) was plotted against the ratio of the branch diameters (\(d_2/d_1\)) and compared with the curves representing the optimum relation. The curves representing the theoretically optimum relation of these parameters to minimum vessel lumen surface and wall shear stress and to minimum vessel lumen volume and pumping power were graphically determined from the model equations by using a computer software system for mathematical applications.\(^7\) The separation of flow streamlines according to the position of the flow divider at the bifurcations, which is a function of \(\gamma\), was calculated for each bifurcation and compared with the division of flow required by the branch caliper, which was defined as (\(d_2/d_1\))^3.

Results

The parameters \(\gamma\) and \(d_2/d_1\), and the division of flow required by the branch caliper were found to scatter considerably around the optimum curves (Fig 2) compared with the separation of flow streamlines according to the flow divider (Fig 3). However, the data points are probably a random sample from a normal distribution around the optimum, as suggested by the \(\chi^2\) test (.40<\(P<.50\). The mean square error of the measured \(\gamma\) and \(d_2/d_1\), and of the estimated flow divisions is relatively low (0.11 and 0.28, respectively). Thus, a trend toward the optimum may be discerned.

A callosomarginal artery was present in 10 of 12 arterial trees analyzed. In these cases the callosomarginal-pericallosal bifurcation, which is the most common aneurysm site at the distal anterior cerebral artery,\(^8\) did not present any particular deviation from the optimum compared with the other branch sites.

Discussion

The apex of bifurcations is the site of maximum hemodynamic stress in a vascular network because of the impact, deflection, and separation of the blood flow streamlines and vortex formation at the lateral angles, which occur despite the stabilizing effect of lowering the Reynolds number (Re) at the branches due to diminishing vessel caliber and blood flow velocity and increase of total luminal area. Arterial bifurcations are known to lower the critical Re at which transition to turbulent flow occurs. This effect is angle-dependent, as with larger branch angles the critical Re becomes even lower.\(^9\) The optimum local condition for the division of flow at a bifurcation in terms of flow orderliness would be the one where the apex is situated such that the impinging streamline corresponds to the division of flow given by the flow rate at the branch vessels.

Evidence indicates that blood flow–induced shear stress plays an important role in the regulation of vascular caliber. In laminar flow the shear stress acting on each unit area along the endothelium is \(\tau=(4\eta\eta/\pi r^3)^{-1}\), where \(\eta\) is blood viscosity. The magnitude of wall shear stress is the
same at every segment of a vascular network obeying the principle of minimum work, because the flow rate influences the shear stress proportionally to the third power of the vessel radius. Every local change of a vascular network, either of volume (surgical resection, body growth or atrophy) or of blood flow (changes in metabolic demand or increased blood flow provoked by an arteriovenous fistula), results in a corresponding change of the drag forces generated by the bloodstream that propagates through all the ramifications of the network. Increase or decrease of the wall shear stress imposes a reorganization of the system to restore an optimum relation between vessel caliber and blood flow, ranging from ephemeral single vessel responses to integrated remodeling of whole vascular networks. Remodeling occurs in the form of increase or decrease of the vessel diameter. Such a mechanism has been observed in the remodeling of cerebral vasculature of mice during development, as wall shear stress at arteriolar branch points was found to be matched on average in neonates and adults. As vessel caliber follows the blood flow, probably the position of the flow divider is also automatically adjusted to the flow-imposed drag forces.

Despite its simplicity, the model used in this analysis shows that an arterial network can be optimum both globally, considering the vascular system as a whole, and locally, considering the position of the flow divider. The model was proposed by Zämir in 1982. The same author compared the position of the apex in relation to the ostium of the parent artery (\( \gamma \)) and the ratio of the branch diameters (\( d_2/d_1 \)) measured in 137 bifurcations of different parts of the vasculature of humans, rabbit, rat, and pig. The study did not include intracranial arteries. No discernable difference was found in the relation of these parameters in different organs and species. The data points were found to scatter around the optimum curves with a clear trend toward the optimum. Zämir considered his results “consistent with a position of the apex which is optimum from the point of view of local hemodynamics,” ascribing the deviations from the optimum to simplifying features inherent in modeling biological systems. Fig 2 presents our data analyzed in a similar manner. To evaluate the relation between these parameters and the division of flow, we additionally calculated the theoretical separation of the flow in the parent artery as a function of \( \gamma \) (Fig 3).

Much of the scatter can be ascribed to dynamic changes of the bifurcations during the cardiac cycle. Changes in shape and luminal cross-sectional area were observed at the bifurcations of isolated human cerebral arteries in response to changes in static distending transmural pressure. Such changes are suggested to occur in vivo. If present, they can modify the local flow patterns at the flow divider within each cardiac cycle. A theoretical model recently proposed to explain the formation of saccular aneurysms is based on the absence of static equilibrium at the apex of the bifurcations as described above. Changes of the branch angles and in the apex position provoked by pulsatile pressure waves during the cardiac cycle could lead to fatigue and disruption of the vessel wall elements and aneurysm formation. Whether the small dynamic changes of the branching geometry stabilize continuously the blood flow or increase locally the disturbance of laminar flow, consequently increasing locally the wall shear stress, is at present unclear.

We conclude that the bifurcations of the cerebral arteries distal to the circle of Willis appear to be optimized to avoid increased hemodynamic stresses both globally (blood flow/vessel radius relation) and locally (flow divider position) in the same manner as in extracranial arteries. Intracranial and extracranial arteries are continually exposed to changes in flow and perfusion pressure during every cardiac cycle and in

**Fig 2.** Position of bifurcation apex in relation to ostium of parent artery (\( \gamma \) or \( c_2/c_1 \)) plotted against ratio of branch diameters (\( d_2/d_1 \)) and compared with curves representing the theoretically optimum relation of these parameters to minimum vessel lumen surface and wall shear stress (on left) and to minimum vessel lumen volume and pumping power (on right) according to model. Data points have a mean square error of 0.11 to area between curves (optimum region) and are probably a sample from normal distribution around optimum (.40 < P < .50).

**Fig 3.** Division of flow required in branches according to diameter exponent plotted against theoretical values of \( I_2(\gamma)/I_1(\gamma) \) given by model. Diagonal line represents division of flow when \( (r_2/r_1)^3 = I_2(\gamma)/I_1(\gamma) \). Mean square error to the optimum is 0.28, and points probably have normal distribution around optimum (.40 < P < .50).
varying body activities. However, saccular aneurysms are uncommon in the extracranial arteries and are known to occur even in distal cerebral arteries, especially in association with high blood flow conditions such as arteriovenous malformations. The distal anterior cerebral artery, which we chose as representative of the arterial tree distal to the circle of Willis, has been reported as the aneurysm site in 4.4% to 6.1% of all aneurysms and is often associated with multiple aneurysms. The cerebral arteries are probably more liable to aneurysm formation caused by intrinsic characteristics of the vessel wall. They are poorly supported by surrounding tissue, have thinner and stiffer walls than extracranial arteries of the same caliber, lack external elastic lamina, and with increasing age develop defects of the muscular layer and fenestrations of the internal elastic lamina. Moreover, the blood flow at the circle of Willis, which is the most common site of saccular aneurysm formation, is subjected to more variable hemodynamic stresses than those in distal cerebral arteries because of complex blood flow patterns due to anatomical variations and physiological changes of flow provoked by different positions of the head.

References

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